Example 1 : Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm (see Fig. 12.6).

Solution : Here we have perimeter of the triangle = 32 cm, a = 8 cm and b = 11 cm.

Third side c = 32 cm – (8 + 11) cm = 13 cm

So, $2s$ = 32, i.e., s = 16 cm,

   $s – a$ = (16 – 8) cm = 8 cm,

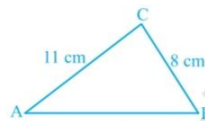 $s – b$ = (16 – 11) cm = 5 cm,

   $s – c$ = (16 – 13) cm = 3 cm.

Fig. 12.6

Therefore, area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{16 \times 8 \times 5 \times 3} \text{ cm}^2 = 8\sqrt{30} \text{ cm}^2$$

Example 2 : A triangular park ABC has sides 120m, 80m and 50m (see Fig. 12.7). A gardener *Dhania* has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of ₹20 per metre leaving a space 3m wide for a gate on one side.

Solution : For finding area of the park, we have

   $2s$ = 50 m + 80 m + 120 m = 250 m.

i.e., s = 125 m

Now, $s – a$ = (125 – 120) m = 5 m,

   $s – b$ = (125 – 80) m = 45 m,

   $s – c$ = (125 – 50) m = 75 m.
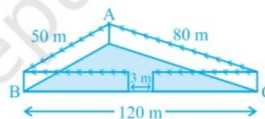

Fig. 12.7

Therefore, area of the park = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{125 \times 5 \times 45 \times 75} \text{ m}^2$$

$$= 375\sqrt{15} \text{ m}^2$$

Also, perimeter of the park = AB + BC + CA = 250 m

Therefore, length of the wire needed for fencing = 250 m – 3 m (to be left for gate)

$$= 247 \text{ m}$$

And so the cost of fencing = ₹20 × 247 = ₹4940

MATHEMATICS

Example 3 : The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 300 m. Find its area.

Solution : Suppose that the sides, in metres, are $3x$, $5x$ and $7x$ (see Fig. 12.8).

Then, we know that $3x + 5x + 7x$ = 300 (perimeter of the triangle)

Therefore, $15x$ = 300, which gives x = 20.

So the sides of the triangle are 3 × 20 m, 5 × 20 m and 7 × 20 m

i.e.,   60 m, 100 m and 140 m.

Can you now find the area [Using Heron's formula]?

Fig. 12.8

We have $s = \dfrac{60 + 100 + 140}{2}$ m = 150 m,

and area will be $\sqrt{150(150-60)(150-100)(150-140)}$ m²

$$= \sqrt{150 \times 90 \times 50 \times 10} \text{ m}^2$$

$$= 1500\sqrt{3} \text{ m}^2$$